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LETTER TO THE EDITOR

The dynamics of the Bean critical state

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Abstract. A simple one-dimensional model to simulate the establishment of the Bean critical state is introduced. It is shown that the dynamics of the flux lines as they enter the superconductor are dominated by 'avalanches'. The distribution of distances moved by vortices in the avalanches obeys a power law with an exponent of -1. This suggests that the Bean state is a self-organized critical state. The density of flux lines is parabolic.

The motion of flux lines in a type II superconductor results in phase slip and hence dissipation, since a potential difference is required to maintain the phase difference. If a current is passed through a superconductor a non-uniform distribution of flux density will be established. To prevent the flux lines from sliding, and hence causing dissipation, the flux lines must be pinned by crystalline defects or random inhomogeneities. If, however, the magnetic pressure gradient exceeds the pinning force movement of flux lines arises [1]. This defines the critical current density, j_c. An understanding of the onset of flux motion is therefore important in understanding the I–V characteristics of type II superconductors. This in turn depends on an understanding of the non-equilibrium distribution of flux line density, and the manner in which it is established.

The Bean critical state describes the metastable distribution of flux density in a dirty superconductor [2]. Observation of $1/f$ noise when this state is perturbed by external magnetic fields [3, 4] leads to the suggestion that this critical state is in fact a self-organized critical (SOC) state, that is one which is established dynamically in a dissipative system and which is always on the brink of an instability [5]. A SOC state is also characterized by having no intrinsic length and time scales.

To investigate the dynamics of the Bean critical state we have devised a simple one-dimensional simulation. In this simulation we have in mind the movement of flux lines into a semi-infinite superconductor as an external magnetic field is turned on. We envisage flux lines, which are nucleated on the surface of the superconductor by the surface current, being driven into the superconductor by their mutual repulsive interactions. A random distribution of pinning centres pins the vortices as they propagate through the superconductor, hence establishing the Bean critical state.

Let us now describe the model in more detail. Rather than using the correct inter-vortex potential, namely $K_0(r_{ij}/\lambda)$, we adopt a simplified short-range version introduced by Pla and Nori [6] in the interests of numerical simplicity. In particular, the potential experienced by a vortex at x_i from the other vortices is

$$V_{vi} = \sum_{j \neq i} A_v \left(|x_i - x_j| - \xi_v\right)^2 \qquad \text{for } |x_i - x_j| \leqslant \xi_v. \tag{1}$$

The mutual force between the ith and jth vortex, $f_{ij} = -\partial V_i/\partial x_{ij}$, therefore has a range ξ_v and decreases linearly with separation.

The distribution of pinning centres, $\{x_\alpha\}$, is taken to be pseudo-random, that is the pinning centre interactions do not overlap and there is a maximum permissible separation distance. We assumed two types of interaction between the pinning centres and the vortices: a finite range Hooke's law

$$V_{pi} = \sum_\alpha -\frac{\xi_p}{2} + \frac{1}{2\xi_p}(x_i - x_\alpha)^2 \qquad \text{for } |x_i - x_\alpha| \leqslant \xi_p \tag{2a}$$

and a sinusoidal dependence,

$$V_{pi} = \sum_\alpha -\frac{\xi_p}{\pi}\left[1 + \cos\left(\pi(x_i - x_\alpha)/\xi_p\right)\right] \qquad \text{for } |x_i - x_\alpha| \leqslant \xi_p. \tag{2b}$$

At the surface of the superconductor, $x = 0$, there is a force, F, of range ξ_v driving in the vortices.

It is convenient to introduce a dimensionless interaction strength, η, as the ratio of the work done in removing a vortex from a pinning centre to the work done in bringing two vortices together, namely,

$$\eta = \int_0^\infty f_{i\alpha}(x_{i\alpha})\mathrm{d}x_{i\alpha} \bigg/ \int_\infty^0 f_{ij}(x_{ij})\mathrm{d}x_{ij}. \tag{3}$$

For the vortex-pinning centre interaction (2a) this is, $(\xi_p/2)/(A_v\xi_v/2)$ while for the interaction (2b) it is, $(\xi_p/\pi)/(A_v\xi_v/2)$. Values of $\eta \ll 1$, $\simeq 1$, and $\gg 1$ represent weak, intermediate to strong and very strong pinning, respectively. We work in the intermediate to strong pinning regime, scale all lengths by ξ_v and take $\xi_p = \xi_v$.

Let us now describe the simulation. The driving force, F, is turned on slowly from zero in increments of 0.01. Initially there are no vortices in the superconductor. At each increment of F vortices are allowed to enter the superconductor and attain their equilibrium positions. This is determined by the net force on each vortex, resulting from the other vortices and the pinning centres, being zero. As the vortices enter the superconductor they shunt other vortices further along, thereby building up a distribution of flux line density [7]. By the nature of this one dimensional simulation each vortex usually has (at least) two neighbours with which it is interacting. Hence, if a vortex enters the superconductor all the vortices will move forward. Typically there is 'stick-slip' behaviour: as F is increased vortices will be unable to enter the superconductor owing to the outward pressure from the pinned vortices. However, for a critical value of F this outward force will be overcome and vortices will flood in causing an 'avalanche' of flux motion. It is important to note that these 'avalanches' usually cause all the vortices to move. Hence, there is no distribution of avalanche sizes as one expects in a two- or three-dimensional simulation. However, there is a distribution of the distances moved by the vortices which shows scaling behaviour [8]. In figure 1(a) we plot the distribution of vortices moving a distance d for the interaction (2a) with $\eta = 1$. To obtain this plot the force has been increased from 0 to 3 in steps of 0.01. For each increment of F the number of vortices moving a distance $d \to d + \Delta d$ is recorded. The figure represents the integrated distribution up to $F = 3$, at which point 192 vortices have entered the superconductor. The slope is -1.00 over three orders of magnitude. Very similar results were found for the interaction (2b), with $\eta = 2/\pi$, shown in figure 1(b). In

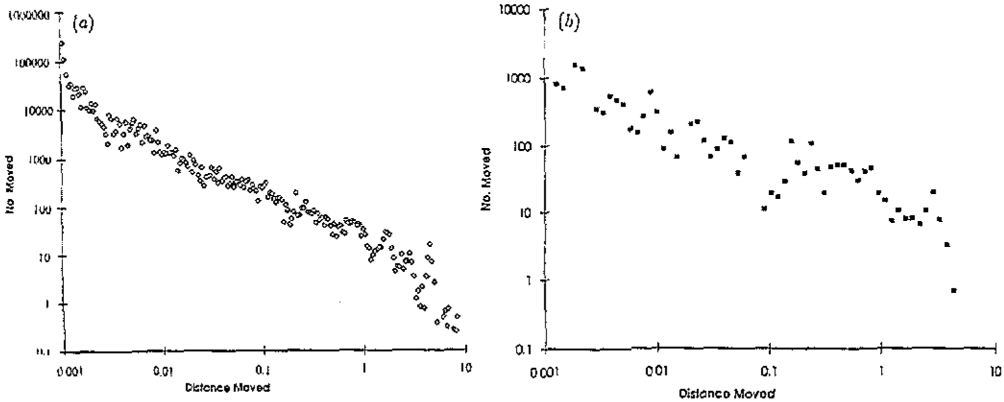

Figure 1. (*a*) The number of vortices (arbitrarily normalized) moving a distance d versus *d* for the interaction (2*a*). The distance is scaled by ξ_v. The external force, F, has been increased from 0 to 3 in increments of 0.01, with a pinning strength, $\eta = 1$. (*b*) The same as (*a*) with the interaction (1*b*). F has been increased from 0 to 1 in increments of 0.001 and $\eta = 2/\pi$.

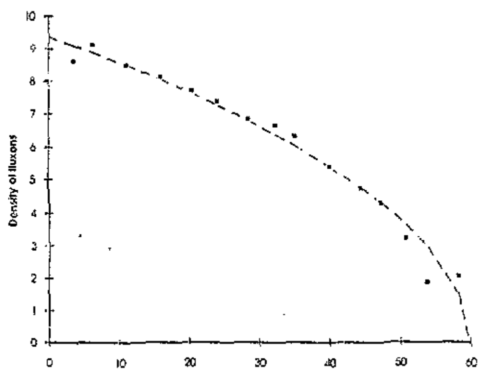

Figure 2. The flux density for 192 vortices using the interaction (2*a*). The squares represent the average density at a pinning centre, averaged over neighbouring pinning centres. The dashed line is a parabolic fit to the data.

this case F was increased from 0 to 1 in increments of 0.001. In the best linear region, from $d = 0.001$ to 0.1, the slope is close to -1.

We next consider the density of flux lines after 192 have entered the superconductor. This is shown in figure 2 where the squares represent the density of vortices at a pinning centre averaged over neighbouring pinning centres. The dashed line is a parabolic fit to the data. It is revealing to note that this density distribution is precisely what one would predict in the Bean critical state if the average pinning force is independent of local flux density. To see this we recall that the Bean critical state is specified when the force from the pinning centres balances the magnetic force arising from the gradient in the magnetic pressure [1], namely,

$$\left|\frac{B}{4\pi}\frac{\mathrm{d}B}{\mathrm{d}x}\right| = f_c$$

in one dimension. If f_c is assumed constant, this is easily integrated to give a magnetic profile of

$$B(x) = B(0)\,(1 - x/\Lambda)^{1/2}$$

where $\Lambda = 8\pi f_c B(0)^2$.

In conclusion, we have introduced a simple one-dimensional model to simulate the establishment of the Bean critical state. We showed that the dynamics of the flux lines as they enter the superconductor is dominated by 'avalanches'. The distribution of distances moved by vortices in the avalanches follows a power law behaviour with an exponent of -1. This would suggest that the density distribution of flux lines is a self-organized critical state. Finally, we found that the density of flux lines follows a parabolic behaviour, valid in the Bean model with a constant pinning force.

There are various ways in which this model is deficient. Most obviously, it is a one-dimensional simulation, so the flux lines always have two neighbours and disturbances propogate through the entire system. It also means that flux lines cannot slide pass one another. Secondly, the correct long-range inter-vortex potential was not used. Such a long range potential means that in practice each vortex interacts directly with many other vortices, leading to the concept of the vortex bundle [9]. This may effect the validity of the self-organized criticality picture.

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Note added in proof. 'Stick-slip' behaviour has recently been observed in the Bean critical state for single, untwinned crystals of YBaCuO [10].

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